

Combinatorial Design of Rigidly Folding Quad-Meshes and Multishape Metamaterials

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Abstract

The simplest foldable origami structure is the 4-vertex, which can be folded along two distinct, nonlinear folding motions [Waitukaitis et al, PRL 2016]. This multibranch behavior provides a unique opportunity to create multishape reconfigurable quadrilateral metamaterials. However, the design of rigidly folding origami-based metamaterials is hard: random tilings of 4-vertices do not fold, as they generically do not satisfy loop-conditions that ensure that all connected 4-vertices can fold simultaneously. As a result, only two types of rigidly quadrilateral meshes are known. First, for flat-foldable 4-vertices, numerical methods allow to find crease patterns that satisfy these the loop conditions; however, these only allow for a single folding motion. Second, a few highly symmetric and periodic quadrilateral meshes which do feature multiple folding branches are known.

Here we set out to systematically design and characterize rigidly folding quadrilateral patterns which have multiple folding branches. We introduce a combinatorial strategy that allows the rational design of an infinite number of rigidly folding quadrilateral meshes with multiple folding branches. To do so, we leverage a yet unexplored symmetry between the folding motions of an arbitrary 4-vertex and its supplement. Combining a 4-vertex, its mirror image, and their 2 respective supplemented vertices, we obtain combinatorial rules that yield 148 distinct 2×2 patterns that rigidly fold (so called Kokotsakis meshes). These 2×2 patterns can be represented by two-dimensional tiles, and the problem of finding larger folding patterns can be translated to the tiling of these 148 tiles.

We show how to completely solve this tiling problem, which leads to four distinct classes of folding patterns. All known simple quadrilateral meshes are contained within these classes. The resulting designs are rigidly foldable but need not be symmetric or periodic, and their multitude grows exponentially with system size – we precisely determine their multitude.

All our designs leverage the bi-branch nature of individual vertices to yield multi branch origami patterns, and we obtain precise results for the number of branches as function of system size for each distinct class of patterns.

Finally we use our results to rationally design the crease pattern of a sheet that can rigidly be folded into two shapes, representing the letters α and ω . We use laser cutting to realize these

designs and demonstrate that their two distinct folded shapes closely resemble the design target. Thus, we demonstrate a combinatorial strategy for the rational design of pluripotent origami-based metamaterials.

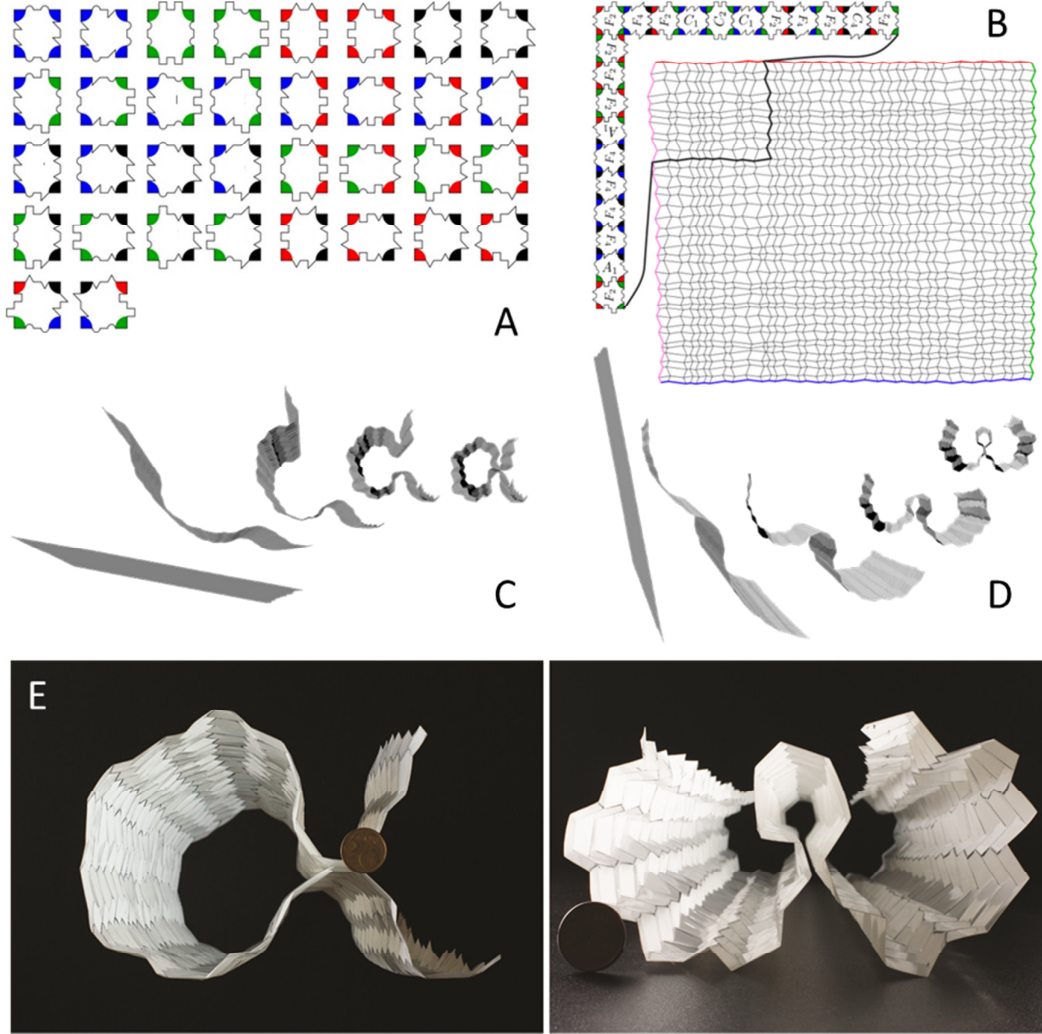


Figure 1: Rational Design of Multi-branch Rigidly Folding Quadrilateral Meshes. A: Tiles representing rigidly folding 2x2 Origami meshes. B: Combinatorial design of large folding mesh. C-D: Rigid folding of this mesh along its two folding branches. E: Experimental Realization.